A Fast Technique for Analysis of Waveguides

Piotr Przybyszewski, Jacek Mielewski, and Michał Mrozowski, *Member, IEEE*

***Abstract*— This letter presents a new technique for fast calculation of dispersion characteristics in inhomogeneously loaded waveguides. The method calculates an approximate value of propagation constant at a desired frequency based on more accurate computations of the field distribution and propagation constant at a few selected frequency points. Numerical tests show that the relative error in propagation constant below 0.05% can easily be obtained throughout a very wide frequency band for a dominant mode using accurate data calculated at as few as four frequency points.**

***Index Terms*— Finite-difference methods, loaded waveguides, moments methods.**

I. Introduction

Calculation of dispersion characteristics of waveguides is one of the fundamental tasks in computational electromagnetics. Traditionally, at least in frequency domain, the wide-band characterization of a waveguiding structure requires repetitive numerical solution of a boundary value problem at successive frequency points. If the computational cost of each solution is high, the overall time required to complete the computation for all points of interest may be long. Recently, two different approaches have been proposed to circumvent this problem. One solution [1], [2] expands the fields at arbitrary frequency using the eigenmodes at cutoff as a basis. The second approach [3] employs a technique called the asymptotic waveform evaluation (AWE) that uses the Taylor series or Pade approximation to represent the dispersion characteristics around a selected frequency point. In this letter we propose a yet another method which calculates an approximate value of propagation constant at a desired frequency based on more accurate computations of the field distribution and propagation constant at a few selected frequency points.

II. Analysis

Suppose that the problem to be analyzed can be written in the following operator form

$$\mathbf{L}u + \omega^2 u - \beta^2 \mathbf{S}u = 0 \quad (1)$$

where u, β are the unknown field and propagation constant, ω is the angular frequency, and $\mathbf{L}$ and $\mathbf{S}$ are operators derived from Maxwell's equations. Let us assume that the above problem has been solved at N discrete points on the frequency axis so that we know triads $\{\omega_i^2, \beta_i^2, u_i(\omega_i, \beta_i)\}, i = 1 \cdots N$ which satisfy equation

$$\mathbf{L}u_i + \omega_i^2 u_i - \beta_i^2 \mathbf{S}u_i = 0 \quad (2)$$

as well as $\{\omega_i^2, \beta_i^{*2}, v_{i*}(\omega_i, \beta_i^*)\}, i = 1 \cdots N$ which solve the transposed equation for β_i^{*2}. Outside these points, we approximate the solution in the following way:

$$u(\omega, \beta) = \sum_{i=1}^{N} a_i u_i \quad (3)$$

with $a_i = a_i(\omega, \beta)$. Substituting the above decomposition into (1) and simultaneously adding and subtracting $\sum a_i \omega_i^2 u_i$ one gets

$$\sum_{i=1}^{N} a_i[\mathbf{L}u_i + \omega_i^2 u_i + (\omega^2 - \omega_i^2)u_i - \beta^2 \mathbf{S}u_i] = 0. \quad (4)$$

Using (2) we can now replace the first two terms under the summation sign with $\beta_i^2 \mathbf{S}u_i$. Accordingly, the equation to be solved becomes

$$\sum_{i=1}^{N} a_i[(\beta_i - \beta^2)\mathbf{S}u_i + (\omega^2 - \omega_i^2)u_i] = 0. \quad (5)$$

This equation is converted to a set of linear equations by taking the inner product of (5) with the eigenfunctions v_{i*} which satisfy the equation transposed to (2). This procedure gives a generalized eigenvalue problem in the form

$$[\underline{\underline{G}}(\omega^2 \underline{\underline{I}} - \underline{\underline{\Omega}}^2) + \underline{\underline{S}}\,\underline{\underline{Z}}^2]\underline{a} - \beta^2 \underline{\underline{S}}\,\underline{a} = 0 \quad (6)$$

where $\underline{\underline{\Omega}} = \mathrm{diag}[\omega_i^2]$, $\underline{\underline{Z}}^2 = \mathrm{diag}[\beta_i^2]$, $\underline{a} = [a_1, a_2 \cdots a_N]^T$, and the elements of matrices $\underline{\underline{G}}$ and $\underline{\underline{S}}$ are given by $G_{ki} = \langle u_i, v_{k*}\rangle$ and $S_{ki} = \langle \mathbf{S}u_i, v_{k*}\rangle$, respectively, where $\langle \cdot, \cdot \rangle$ denotes an inner product. Solving the above problem for $\beta^2(\omega)$ and $\underline{a}(\omega, \beta)$ one gets the approximate dispersion characteristics and field distribution for up to N modes of a waveguide of interest. In practice, one often is interested in the dispersion characteristics of a particular mode. In that case, the basis $\{\omega_i^2, \beta_i^2, u_i(\omega_i, \beta_i)\}, i = 1 \cdots N$ consists of the solutions evaluated for this particular mode at different frequency points. This choice gives a better approximation of the mode of interest but increases the error for the remaining $N - 1$ modes. Note, that if the number of expansion terms N is small, the solution of eigenvalue problem (6) is very fast.

In order to illustrate how this new technique can be used in practice let us consider a closed waveguide inhomogeneously loaded with an isotropic dielectric medium. The wave equation for such a structure is given by [4]

$$-\nabla_t \times \mu^{-1}\nabla_t \times \epsilon^{-1}\vec{D}_t + \mu^{-1}\nabla_t \epsilon^{-1}\nabla_t \cdot \vec{D}_t + \omega^2 \vec{D}_t - \beta^2 \mu^{-1}\epsilon^{-1}\vec{D}_t = 0 \quad (7)$$

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The authors are with the Department of Electronics, Telecommunications and Informatics, Technical University of Gdańsk, 80-952 Gdańsk, Poland (e-mail: pip@pg.gda.p; jamie@eti.pg.gda.pl; mim@pg.gda.pl).

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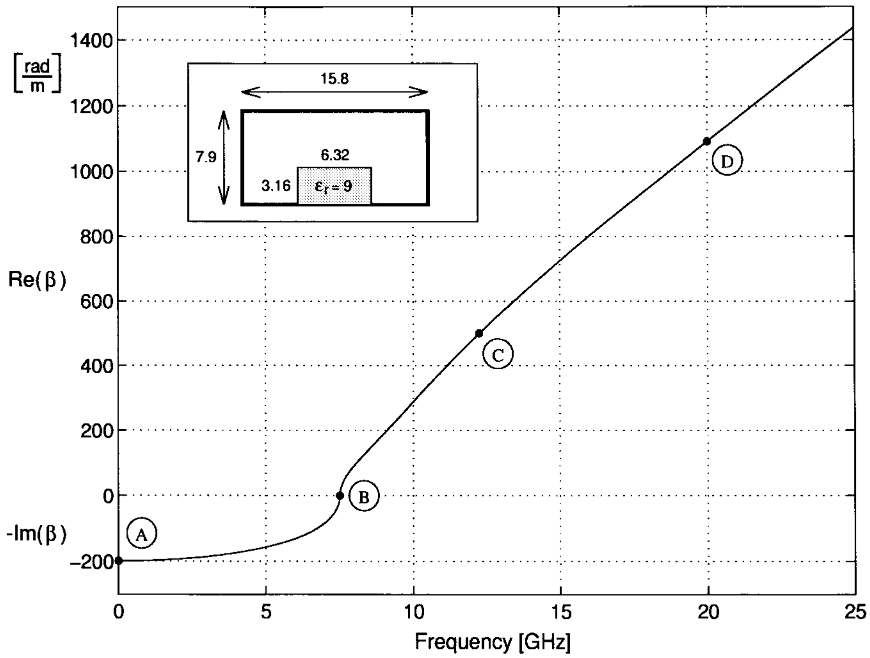

Fig. 1. Dispersion characteristic of a dominant mode in a rectangular image guide calculated using the FDFD method. Symbols A–D indicate four points at which modal fields have been calculated and used as a basis in a fast algorithm described in this letter.

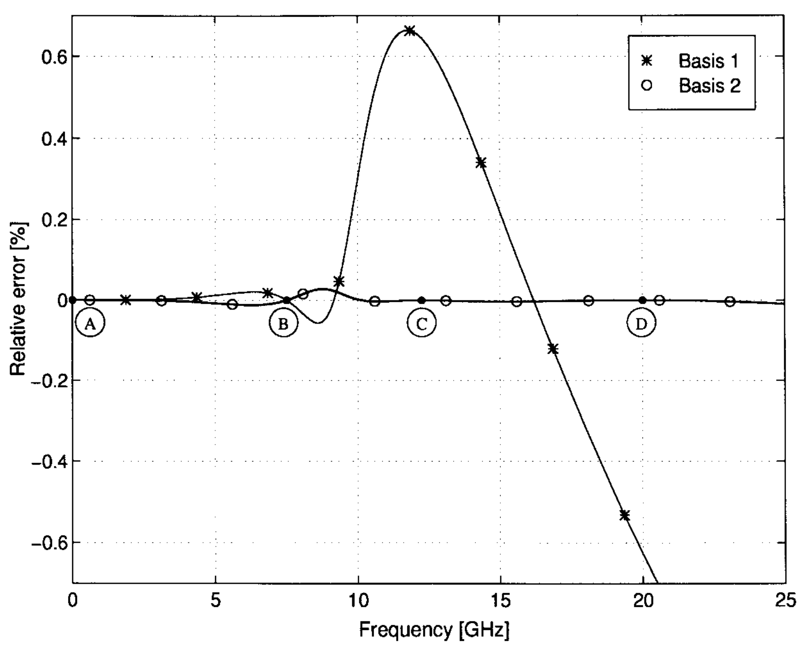

Fig. 2. Error in propagation constant (relative to FDFD computations shown in Fig. 1) in a fast algorithm with fields evaluated at points A and B (Basis 1) or A, B, C, D (Basis 2) used as a basis in expansion (3).

where $\vec{D}_t$ is the transverse electric flux density while μ, ϵ are the (absolute) permeability and permittivity of the medium, respectively. If the medium is lossless and the guide walls are perfectly conducting, the equation transposed to (7) is fulfilled by $\hat{z} \times \vec{B}_{t*}$ [4], where $\hat{z}$ is a unit vector in the z direction and $\vec{B}_{t*}$ is the transverse magnetic flux density. (The asterisks in the subscript denotes that a field corresponding to β_i^{*2} is substituted. For noncomplex modes $\vec{B}_{t*} = \vec{B}_t$, otherwise $\vec{B}_{t*} = \vec{B}_t^*$.) Comparing (7) with (1) it is seen that $\mathbf{S} = (\mu\epsilon)^{-1}$. Accordingly, the elements of matrices $\underline{\underline{G}}$ and $\underline{\underline{S}}$ can easily be evaluated as follows:

$$G_{ki} = \langle u_i,\, v_{k*}\rangle = \hat{z} \cdot \int_\Omega \vec{D}_{ti} \times \vec{B}^*_{t_{k*}}\, d\Omega \qquad (8)$$

$$S_{ki} = \langle \mathbf{S}u_i,\, v_{k*}\rangle = \hat{z} \cdot \int_\Omega \vec{E}_{ti} \times \vec{H}^*_{t_{k*}}\, d\Omega \qquad (9)$$

where Ω denotes the cross section of a guide.

III. Results

The technique described in the preceding section has been tested on several structures. In this letter we shall give the

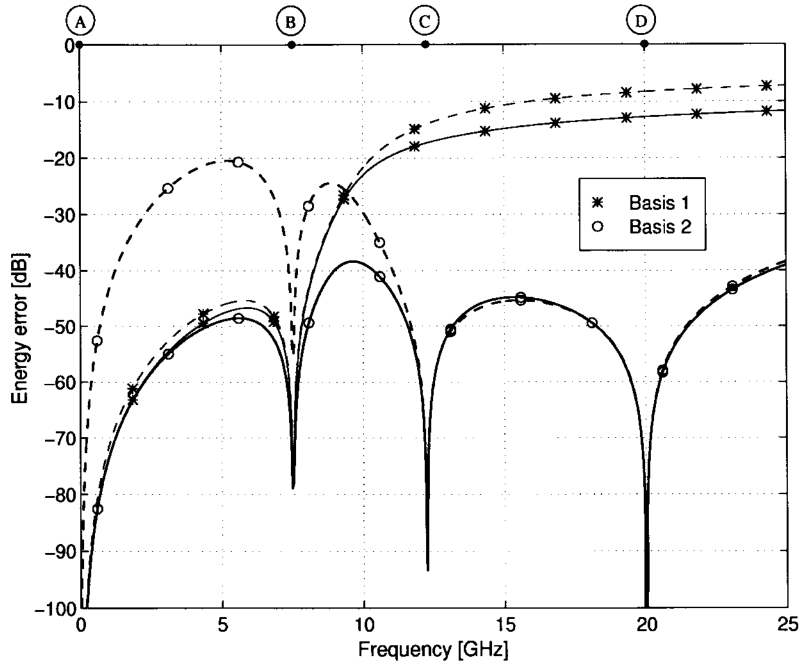

Fig. 3. Electric (solid line) and magnetic (dashed line) field energy error (relative to FDFD computations shown in Fig. 1) in a fast algorithm with fields evaluated at points A and B (Basis 1) or A, B, C, D (Basis 2) used as a basis in expansion (3).

results for the dominant mode of an image guide shown in Fig. 1.

In order to be able to evaluate the accuracy of the present method the complete dispersion characteristic and the modal fields for the dominant mode have been found using the finite-difference frequency-domain (FDFD) with a grid of 40×20 points. This implied solving a sparse eigenvalue problems with a matrix of the order of 1580 for all points within frequency range. Subsequently, the new algorithm was applied with two different sets of points used as a basis. (These points are denoted as A–D in Figs. 1–3.) In the first test the basis consisted of fields computed at $f = 0$ and at cutoff (points A, B). In the second test two additional points (C, D) were used. This implies that the sparse solver had to be applied only two (test 1) or four times (test 2). Using the fields computed at these points two small eigenvalue problems were constructed and solved at the same points at which the reference solution was computed with the FDFD method. Since this time the problems were very small (2×2 for Basis 1 and 4×4 for Basis 2) the computational workload of this step was marginal. Figs. 2 and 3 show the errors in propagation constant and the electric and magnetic field of the new algorithm relative to the reference FDFD solution. It is seen that even for the smallest basis consisting of just two points the new algorithm gives very good results below cutoff and may be regarded as quite satisfactory (error in propagation constant below 1%) also above cutoff. Adding two more points to the basis pushes the relative error in propagation constant below 0.05% for all points within the region of interest.

IV. Conclusions

A new technique for the fast analysis of waveguides has been developed. The technique has a hybrid character and consists of two steps. In the first step the eigenvalue problem is solved for the propagation constant and modal field distribution at a few discrete frequency points using an arbitrary numerical or analytical technique. Subsequently, a new small matrix eigenvalue problem is constructed using these solutions as a basis. This new problem yields the propagation constant and field distributions for all frequency points of interest.

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